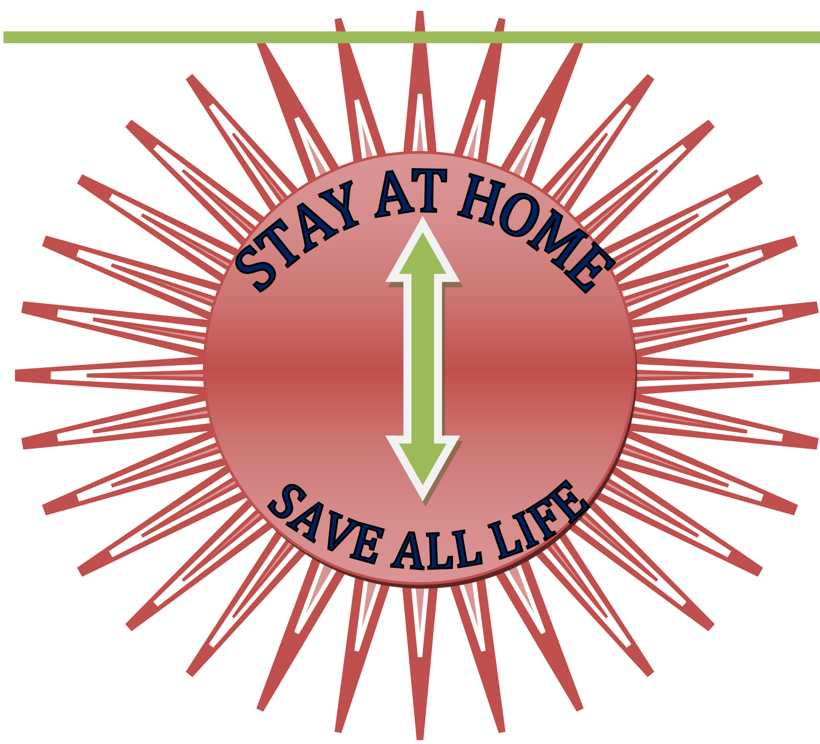
AL-HAFEEZ COLLEGE,ARA

ONLINE CLASSES (PDF MODE)

MATHEMATICS LECTURE NO- 02



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Hi dear students as we discussed about some properties of sets, now we shall start further concepts.

De Morgan's Theorem

First Form : To prove that (a) $(A \cup B)' = A' \cap B'$ i.e., the complement of union of two sets is the intersection of their complements.

Here A' has been used as a complement set of A etc.

Proof : Let $x \in (A \cup B)'$.

Then $x \notin A \cup B$ (by definition)

This $\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in (A' \cap B')$.

Thus $(A \cup B)' \subseteq A' \cap B'$ ...(1)

Again suppose that $y \in A' \cap B'$.

Then $y \in A'$ and $y \in B'$ i.e., $y \notin A$ and $y \notin B$.

This $\Rightarrow y \notin A \cup B \Rightarrow y \in (A \cup B)'$

$$\therefore \quad A' \cap B' \subseteq (A \cup B)' \qquad \ldots(2)$$

From (1) and (2), we get

$$(A \cup B)' = A' \cap B'.$$ proved

To prorve that $(A \cap B)' = A' \cup B'$

i.e., the complement of intersection of two sets is the union of their complements.

Proof : Let $x \in (A \cap B)'$.

This $\Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ or $y \notin B$

$\Rightarrow x \in A'$ or $y \in B'$

$\Rightarrow x \in A' \cup B'$. by def

Thus $(A \cap B)' \subseteq A' \cup B'$. ...(1)

Again, let $y \in A' \cup B'$.

This $\Rightarrow y \in A'$ or $y \in B'$

$\Rightarrow y \notin A$ or $y \notin B$

$\Rightarrow y \notin (A \cap B)$

$\Rightarrow y \in (A \cap B)'$.

Thus $A' \cup B' \subseteq (A \cap B)'$ ...(2)

From (1) and (2), we get

$$(A \cap B)' = A' \cup B'.$$ proved

De Morgan's theorem in general form

To prove that (i) $\left\{\bigcup_{i=1}^{n} A_i\right\}' = \bigcap_{i=1}^{n} A_i'$

(ii) $\left\{\bigcap_{i=1}^{n} A_i\right\}' = \bigcup_{i=1}^{n} A_i'$

where *i* stands for complement.

Proof: We prove the theorem by using methematical induction.

We shall first of all, prove that

(i) $(A_1 \cup A_2 \cup A_3)' = A_1' \cap A_2' \cap A_3'$

(ii) $(A_1 \cap A_2 \cap A_3)' = A_1' \cup A_2' \cup A_3'$

(i) Let $A_2 \cup A_3 = S$.

Then $(A_1 \cup A_2 \cup A_3)' = (A_1 \cup S)'$

$= A_1' \cap S'$

$= A_1' \cap (A_2 \cup A_3)'$

$= A_1' \cap (A_2' \cap A_3')$

$= A_1' \cap A_2' \cap A_3'$.

Similarly (ii) can be proved.

We prove the first. We assume that (i) is true for $n = m$,

i.e., $(A_1 \cup A_2 \cup A_3 \ldots \cup A_m)'$

$= A_1' \cap A_2' \cap A_3' \cap \ldots \cap A_m'$ ... (1)

Now $\{(A_1 \cup A_2 \cup A_3 .. \cup A_m) \cup A_{m+1}\}'$

$= (A_1 \cup A_2 \cup A_3 \ldots \cup A_m)' \cap A_{m+1}'$

$= (A_1' \cap A_2' \cap A_3' \ldots \cap A_m') \cap A_{m+1}'$; from (1)

Hence (i) is true for $n = m + 1$ also.

But we know that (1) is true for $n = 2, n = 3, \ldots$ and hence, by mathematical induction, it is true for every finite n. This proves (i).

Similarly (ii) is true for any finite n.

General method : Prove that

(i) $X - \bigcup_i A_i = \bigcap_i (X - A_i)$

(ii) $X - \bigcap_i A_i = \bigcup_i (X - A_i)$

Proof : (i) Let $x \in X - \bigcup_i A_i$.

Then $x \in X - \bigcup_i A_i \Rightarrow x \in X$ but $x \notin \bigcup_i A_i$

$\Rightarrow x \in X$ and $x \notin$ any A_i

$\Rightarrow x \in X - A_i$, for each i

$\Rightarrow x \in \bigcap_i (X - A_i)$

Hence $X - \bigcup_i A_i \subseteq \bigcap_i (X - A_i)$ ... (1)

Again, let $y \in \bigcap_i (X - A_i)$.

Then $y \in \bigcap_i (X - A_i) \Rightarrow y \in X - A_i$ for each i

$\Rightarrow y \in X$ and $y \notin$ any A_i

Hence $\bigcap_i (X - A_i) \subseteq X - \bigcup_i A_i$ ... (2)

Combining (1) and (2), we get

$$X - \bigcup_i A_i = \bigcap_i (X - A_i).$$

This proves (i).

(ii) Let $x \in X - \bigcap_i A_i$.

Then $x \in X - \bigcap_i A_i \Rightarrow x \in X$ but $x \notin \bigcap_i A_i$

$\Rightarrow x \in X$ and $x \notin A_i$ for at least one $i \in I$

$\Rightarrow x \in (X - A_i)$ for at least one $i \in I$

$\Rightarrow x \in \bigcup_i (X - A_i)$

Hence $X - \bigcap_i A_i \subseteq \bigcup_i (X - A_i)$ ... (3)

Again, let $y \in \bigcup_i (X - A_i)$.

Then $y \in \bigcup_i (X - A_i) \Rightarrow y \in X$ and $y \notin A_i$ for at least one $i \in I$

$\Rightarrow y \in X$ and $y \notin (\bigcap_i A_i)$

$\Rightarrow y \in X - \bigcap_i A_i$

Hence $\bigcup_i (X - A_i) \subseteq X - \bigcap_i A_i$ ... (4)

Combining (3) and (4), we get

$X - \bigcap_i A_i = \bigcup_i (X - A_i)$ This proves (ii).

If A, B and C are three sets, then

$A \cap (B - C) = (A \cap B) - (A \cap C)$

Proof :

We have, $A \cap (B - C) = A \cap (B \cap C')$

$= (A \cap B) \cap C'$, associative law

$= \phi \cup \{(A \cap B) \cap C'\}$

since $S \cup \phi = S$

$= (A \cap A') \cup \{(A \cap B) \cap C'\}$

$= (A \cap B \cap A') \cup \{(A \cap B) \cap C'\}$
$= (A \cap B) \cap (A' \cup C')$; by distributive law
$= (A \cap B) \cap (A \cap C)'$; by De Morgan's law
$= (A \cap B) - (A \cap C)$.

Second method : Let $x \in A \cap (B - C)$
Then $x \in A \cap (B - C)$
$\Rightarrow x \in A$ and $x \in (B - C)$
$\Rightarrow x \in A$ and ($x \in B$ and $x \notin C$)
$\Rightarrow$ ($x \in A$ and $x \in B$) and ($x \in A$ and $x \notin C$)
$\Rightarrow x \in (A \cap B)$ and $x \notin (A \cap C)$
$\Rightarrow x \in (A \cap B) - (A \cap C)$
$\therefore A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$ ...(1)

Again, let $y \in (A \cap B) - (A \cap C)$.
Then $y \in (A - B) \cap (A - C)$
$\Rightarrow y \in A \cap B$ and $y \notin (A \cap C)$
$\Rightarrow$ ($y \in A$ and $y \in B$) and ($y \notin A$ or $y \notin C$)
$\Rightarrow$ ($y \in A$ and $y \in B$) and $y \notin C$
$\Rightarrow y \in A$ and ($y \in B$ and $y \notin C$)
$\Rightarrow y \in A$ and $y \in (B - C)$
$\Rightarrow y \in A \cap (B - C)$

$\therefore (A \cap B) - (A \cap C) \subseteq A \cap (B - C)$ ...(2)
From (1) and (2), we get $A \cap (B - C) = (A \cap B) - (A \cap C)$

Prove that (i) $P(A \cap B) = P(A) \cap P(B)$

(ii) $P(A \cup B) \neq P(A) \cup P(B)$

Let $X \in P(A \cap B) \Rightarrow X \subseteq A \cap B$

$\Rightarrow X \subseteq A$ and $X \subseteq B$

$\Rightarrow X \in P(A)$ and $X \in P(B)$

$\Rightarrow X \in P(A) \cap P(B)$

$\therefore P(A \cap B) \subseteq P(A) \cap P(B)$ ...(1)

$\Rightarrow Y \in P(A)$ and $Y \in P(B)$

Again, let $Y \in P(A) \cap P(B)$

$\Rightarrow Y \subseteq A$ and $Y \subseteq B$

$\Rightarrow Y \subseteq A \cap B$

$\Rightarrow Y \in P(A \cap B)$ ...(2)

Combining (1) and (2), we get

$P(A \cap B) = P(A) \cap P(B)$

$\Rightarrow X \in P(A)$ or $X \in P(B)$

(ii) Let $X \in P(A) \cup P(B)$

$\Rightarrow X \subseteq A$ or $X \subseteq B$

$\Rightarrow X \subseteq A \cup B$

$\Rightarrow X \in P(A \cup B)$.

$\therefore P(A) \cup P(B) \subseteq P(A \cup B)$.

Hence ,Now we shall study cross product of sets in next class