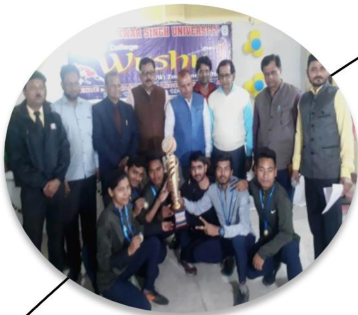

MATHEMATICS LECT. NO- 03

AL-HAFEEZ COLLEGE, ARA

ONLINE CLASSES (PDF MODE)

LECTURES ON SET THEORY FOR B.SC PART 1 (HON'S) 2020- 21

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Ordered pair : An element of the form (a, b) i.e., an element consisting of a pair a and b such that the given element is in a definite order i.e., a is the first element and b is the second element, is called an ordered pair. The word 'ordered' is meant to imply that the order in which the two numbers a and b are written is important. Thus the pair (a, b) is to be considered as different from the pair (b, a).

Cartesian product of two sets

Let A and B be two sets.

Then the set consisting of ordered pairs (a, b) where $a \in A$ and $b \in B$ is called the cartesian product of A and B and is denoted by $A \times B$.

In notations, $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.

Usually $A \times B$ is called the *product set* of A and B and is read as 'A cross B'.

If $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$, calculate $A \times B$ and $B \times A$.

Here, in calculating $A \times B$ we form the ordered pairs by first taking each element of the first set A and then taking each element of the second set B. Similarly in calculating the producet set $B \times A$ we form the ordered pairs by first taking each element of the first set B and then taking each element of the second set A.

Thus, $A \times B = \{1, 3, 5\} \times \{2, 4, 6\}$

$= \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6), (5, 2), (5, 4), (5, 6)\}$

and $B \times A = \{2, 4, 6\} \times \{1, 3, 5\}$

$= \{(2, 1), (2, 3), (2, 5), (4, 1), (4, 3), (4, 5), (6, 1), (6, 3), (6, 5)\}$. Thus $A \times B \neq B \times A$

If $A = \{1, 2,\}$, $B = \{1, 2, 3\}$, find (i) $A \times B$ (ii) $B \times A$

(i) $A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$

(ii) $B \times A = \{1, 2, 3\} \times \{1, 2\}$

$= \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (3, 2)\}$.

Now we have general form:

Cartesian product in general form

Let A, B, C be three given sets.

The product of the three sets A, B, C is denoted by $A \times B \times C$ and is defined by

$$A \times B \times C = \{(a, b, c) \mid a \in A, b \in B, c \in C\}.$$

Here the element (a, b, c) is called ordered triple.

Generalising the result to finite number of sets we have the following definition for the product of n sets.

Let $A_1, A_2, A_3, \ldots A_n$ be n given sets. The set of ordered n-tuples $(a_1, a_2, a_3, \ldots a_n)$, $a_i \in A_i$ for $i = 1, 2, 3, \ldots n$ is called the cartesian product of $A_1, A_2, A_3, \ldots A_n$ and is denoted by $A_1 \times A_2 \times A_3 \times \ldots \times A_n$.

Important

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(i) Let $(x, y) \in A \times (B \cup C)$.

This $\Rightarrow$ that $x \in A$ and $y \in B \cup C$

$\Rightarrow$ $x \in A$ and $\{y \in B$ or $y \in C\}$

$\Rightarrow$ $\{x \in A$ and $y \in B\}$ or $\{x \in A$ and $y \in C\}$;

by distributive law

$$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$$

That is, $(x, y) \in (A \times B)$ or $(x, y) \in (A \times C)$.

Hence $(x, y) \in (A \times B) \cup (A \times C)$

$\therefore \quad A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ ... (1)

Conversely, let $(u, v) \in (A \times B) \cup (A \times C)$.

Then $(u, v) \in A \times B$ or $(u, v) \in A \times C$.

That is, $(u \in A$ and $v \in B)$ or $(u \in A$ and $v \in C)$.

$\Rightarrow \{u \in A\}$ and $\{v \in B$ or $v \in C\}$; by distributive law

$\therefore \; (u, v) \in A \times (B \cup C)$.

Hence $(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ ... (2)

From (1) and (2), we get $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

Now (ii) Let $(x, y) \in A \times (B \cap C)$.

This $\Rightarrow$ that $x \in A$ and $y \in B \cap C$

$\therefore \quad A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ ... (3)

Conversely, let $(u, v) \in (A \times B) \cap (A \times C)$.

Then, $(u, v) \in (A \times B)$ and $(u, v) \in (A \times C)$.

That is, $(u \in A, v \in B)$ and $(u \in A, v \in C)$

i.e., $u \in A$ and $v \in B \cap C$.

This $\Rightarrow \; (u, v) \in A \times (B \cap C)$.

$\therefore \quad (A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$ ... (4)

Now, (3) and (4) together $\Rightarrow$

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

If A, B, C are three sets, prove that Very Imp.

(i) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

(ii) $(A \cap B) \times C = (A \times C) \cap (B \times C)$.

(i) Let $(x, y) \in (A \cup B) \times C$.

Then $x \in (A \cup B), y \in C$

$\Rightarrow \{x \in A \text{ or } x \in B\}$ and $y \in C$

$\Rightarrow \{x \in A, y \in C\}$ or $\{x \in B, y \in C\}$·

by distributive law

$\Rightarrow (x, y) \in (A \times C)$ or $(x, y) \in B \times C$

$\Rightarrow (x, y) \in (A \times C) \cup (B \times C)$.

Hence $(A \cup B) \times C \subseteq (A \times C) \cup (B \times C)$ ... (1)

Again, let $(u, v) \in (A \times C) \cup (B \times C)$

so that $(u, v) \in A \times C$ or $(u, v) \in B \times C$.

This $\Rightarrow \{u \in A, v \in C\}$ or $\{u \in B, v \in C\}$.

Proceding backward from inclusion (1) step-by-step, shall find that $(u, v) \in (A \cup B) \times C$

so that $(A \times C) \cup (B \times C) \subseteq (A \cup B) \times C$ ... (2)

Thus from (1) and (2) we get the equality.

(ii) It can similarly be proved.

OK thank you for attending online PDF mode classes, we will meet again for next lect.