AL- HAFEEZ COLLEGE, ARA

ONLINE CLASSES

MATHEMATICS LECTURE NO- 01

TODAY SUBJECT

SET THEORY

LECTURES ON SET THEORY FOR B.SC PART 1 (HON'S) 2020- 21

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Before going to start we recall some important concept and definition of previous classes which is helpful to understand these new lectures of set theory.

Some Definition on previous concept

sets

A set is a collection of well-defined objects.

Some examples of sets :

(i) The members of Indian Parliament.
(ii) The set of natural numbers.
(iii) The set of points on a line, etc.

The objects belonging to a set are called the elements or nbers of the set.

Sets are usually denoted by capital letters e.g., $A, B, C, D, \ldots X, Y, Z, \ldots$ etc.

and the elements of a set are generally denoted by small letters $a, b, c, d, \ldots, x, y, z$ etc.

The membership of a set is denoted by the symbol $\in$.

Finite and Infinite Sets

A set consisting of finite number of elements i.e., a set whose elements can be put in one-to-one correspondence with the subset of natural numbers viz. $\{1, 2, 3, \ldots n, \ldots\}$ for some n, is called a **finite set.** A set consisting of an infinite number of elements i.e., a set which is not finite, is called **infinite set.**

Ex. (i) $X = \{1, 3, 5, 7, 9\}$ is a finite set.

(ii) $X = \{0, 2, 4, 6, 8, \ldots\}$ is an infinite set.

(iii) The set consisting of all positive integers i.e., $X = \{1, 2, 3, 4, \ldots\}$ is an infinite set.

(iv) The set consisting of all people living in the state of Behar is a finite set.

(v) The set consisting of all days in a week is a finite set.

Singleton set

A set consisting of one element x only is called a **singleton set** and is denoted by $\{x\}$.

Null set (or Empty set)

The set which does not contain any element is called a nul set. The null set is denoted by the symbol ϕ.

Ex. (i) $S = \{x \mid x \text{ is a month having 40 days}\}$ is a null set.

(ii) $S = \{x \mid x^2 + 4 = 0, x \text{ is real}\}$ is a null set.

(iii) $S = \{x \mid x + 5 = 5\}$ is not a null set since solving th equation, we get $x = 0$. So there is an element $x = 0$ in the set S

Subsets

If A and B are two sets such that every element of A is also an element of B, we say that A is a **subset** of B. That is, if $x \in A$ implies ($\Rightarrow$) that $x \in B$, then A is called a subset of B.

The notation for 'is a subset of' or 'is contained in' is $\subseteq$. Thus $A \subseteq B$ means that A is a subset of B or A is contained in B.

Proper subset : If A and B be two sets such that every element of A is also an element of B and if there is at least one element of B that is not in A, we say that A is a **proper subset** of B and we write $A \subset B$.

Ex.1. Let $A = \{1, 2, 4, 6\}$, $B = \{1, 2, 3, 4, 5, 6\}$.

Then $A \subset B$.

Ex.2. Let $N \equiv$ the set of natural numbers i.e., $N = \{1, 2, 3, \ldots\}$

and $Z =$ the set of integers.

Then $N \subset Z$.

Power set

The set which is a collection of all possible subsets of a given set A is called the power set of the set A and it is denoted by $P(A)$ or 2^A.

Symbolically $P(A) = \{X \mid X \subseteq A\}$.

Hence $X \in P(A) \Rightarrow X \subseteq A$ and $X \subseteq A \Rightarrow X \in P(A)$.

Thus if $A = \{a, b, c\}$, then

$$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}.$$

Equality of two sets

If the two sets are equal, i.e., if they have the same elements, then every element of A is an element of B and every element of B is an element of A.

Now, 'every element of A is an element of B' $\Rightarrow A \subseteq B$,

and 'every element of B is an element of A' $\Rightarrow B \subseteq A$.

Consequently, we may define the equality of two sets A and B in the following manner.

The two sets A and B are equal if $A \subseteq B$ and $B \subseteq A$.

This device is very useful in proving the equality of any two sets A and B

Comparability of sets

Let A and B be two sets. If $A \subseteq B$ or $B \subseteq A$, then the sets A and B are said to be *comparable*. Thus if the set A is neither a subset of B and the set B is not a subset of A, then the two sets A and B are said to be incomparable.

Ex. Let $A = \{1, 2, 3\}$ and $B = \{1, 3, 5, 7\}$.

Obviously, A is not a subset of B nor B is a subset of A since $2 \in A$ but $2 \notin B$ and $5, 7 \in B$ but $5, 7 \notin A$.

Therefore the sets A and B are not comparable.

Universal set

In set theory, we often deal with different sets which may possibly be subsets of a fixed set.

Hence that fixed set such that the sets under consideration are subsets is called **universal set.** In other words, a universal set is a fixed set which is a totality of sets under consideration.

Family of sets

If all the elements of a set are sets themselves, then such a set is known as a **family of sets.** The phrase 'class of sets' or 'collection of sets' or 'set of sets' is also used in place of 'family of sets'.

An indexed family of sets

Let I be a fixed non-empty set with elements i and let there be defined a set for every $i \in I$. Then the family of sets A_i where $i \in I$ is called an indexed family of sets and the set I is called an **index set.** Usually the index set I is taken as the set of natural numbers N so that the family of sets $\{A_i : i \in I\}$ can be designated as $\{A_1, A_2, A_3, \ldots A_n, \ldots\}$ and it is known as sequence of sets.

Union of two sets

Definition : *Let A and B be two sets. The union of the sets A and B is the set of all points x which belong to either A or B (or both), that is, either $x \in A$ or $x \in B$.*

The union of A and B is denoted by $A \cup B$.

Thus $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

Ex.1. Let $A = \{1, 2, 3, 5, 8\}$ and $B = \{2, 3, 4, 5\}$.

Then $A \cup B = \{1, 2, 3, 4, 5, 8\}$.

Ex.2. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 3, 4\}$.

Then $A \cup B = \{1, 2, 3, 4, 5\} = A$.

Union of family of sets : Let $\{A_i : i \in I\}$ be a family of subsets of universal set Ω. Then the union of this family of sets is denoted by $\cup\{A_i : i \in I\}$ or $\bigcup\limits_{i \in I} A_i$ or simply by $\cup_i A_i$ and is defined by

$$\cup_i = \{x \mid x \in A_i \text{ for at least one } i \in I\}$$

If the index set I is $N = \{1, 2, 3, \ldots n, \ldots\}$, then the union of the family $\{A_i : i \in N\}$ can be written in the form $\bigcup\limits_{i=1}^{\infty} A_i$.

If the index set I is a subset of N, i.e., if $I = \{1, 2, 3, ..., m\}$, we usually denote $\bigcup\limits_{i=1}^{m} A_i$ by $A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_m$.

Thus if $S_1, S_2, S_3, \ldots S_n$ be any n sets where n is a finite number, then the union of these sets will be a set of all those elements x which will be at least in one of the sets $S_1, S_2, S_3, \ldots S_n$. We write this union as follows :

$$S_1 \cup S_2 \cup S_3 \ldots \cup S_n \text{ or } \bigcup_{n=1}^{n} S_n.$$

Thus $\cup S_n = \{x : x \in S_n \text{ for at least one } n\}$.

Intersection of two sets

Definition : *Let A and B be two sets. The intersection of the sets A and B is the set of all points x which belong to both A and B, that is,* $x \in A$ *and* $x \in B$.

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Thanking you, we will discuss in next lecture no. - 02